CHILDREN'S INFORMAL COMPOSITE AND TRUNCATED PARTITIONING STRATEGIES

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In this paper, we report two subsets of partitioning strategies utilised by a sample of twelve Grade 3 children when they were asked to solve sequences of partitioning problems. These strategies emerged when the children sought to make the process of partitioning more efficient in terms of generating equal shares, reducing the number of steps involved in partitioning and reducing the load on working memory. Implications for instruction are drawn from the findings.

INTRODUCTION

Informal or intuitive knowledge is what learners have prior to instruction, "the applied, real-life, circumstantial knowledge, which can either be correct or incorrect" (Leinhardt, 1988, p. 12). There is growing evidence to suggest that students have well developed informal knowledge of fractions which they are able to use to solve everyday problems (Behr, Harel, Post, & Lesh, 1992; Hiebert, 1988; Leinhardt, 1988; Mack, 1990; Resnick, 1986); including informal knowledge about partitioning and equivalence (Kieren, 1976; Pothier & Sawada, 1983). Research has generally indicated that informal knowledge can serve as a basis for understanding symbols and procedures (Hiebert, 1988). Thus, in recent years, there has been an increased interest in investigating young children's informal strategies for partitioning. These studies have found that young children tend to use a variety of informal strategies when confronted with partitioning problems (Lamon, 1996; Pitkethly & Hunting, 1996; Pothier & Sawada, 1983, Streefland, 1991).

In the study being reported in this paper, we investigated the partitioning strategies utilised by a purposively selected sample (Patton, 1990) of twelve Grade 3 children when they were asked to solve sequences of "realistic" partitive quotient fraction problems. Our purpose for conducting this study was to add to the literature on young children's partitioning strategies by not only setting out to identify new partitioning strategies but also to develop a taxonomy for classifying partitioning strategies in terms of their ability to facilitate the abstraction by young children of the partitive quotient fraction construct. In the partitive quotient fraction construct, the fraction $^x/_y$ refers to the quantity represented by one of the resulting shares when the quantity x is partitioned into y equal parts. For example, if two whole cakes are partitioned into three equal shares, then each share is two thirds of a cake. The major findings from this study are reported in Charles (1998), Charles, Nason, and Cooper (submitted), Charles, Nason, Cooper and McRobbie (submitted), and Charles, Cooper, Nason and McRobbie (in preparation). In this paper, we report on two subsets of partitioning strategies utilised by children in the study: composite and truncated strategies.

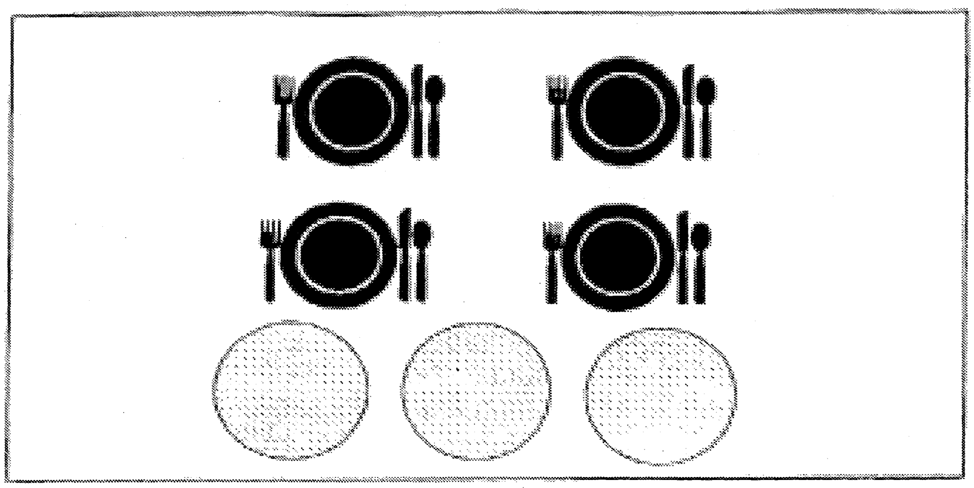
METHOD

The study was couched in the constructivist paradigm. The data gathering technique used was the clinical or mixed method technique of Ginsburg, Kossan, Schwartz, and Swanson (1983) which is a combination of Piaget's clinical interview and the talk aloud procedures of Ericsson and Simon (1984). According to Ginsberg et al., this technique enables researchers to not only elicit complex intellectual activity but also to identify the internal symbolic mechanisms underlying the complex intellectual activity.

Instruments

A set of thirty realistic partitioning problems were developed for the study. In each of the problems, the children were asked to assume the roles of waiters/waitresses serving pizzas, pancakes, pikelets, icecream bars, apple pies, licorice straps to a number of customers sitting at a restaurant table. An example of one of these problems is presented in Figure 1 below.

Figure 1
Partitioning Problem 1(f): 3 pizza among 4 people



Three task variables were taken into consideration during the development of the partitioning problems: (i) types of analog objects, (ii) number of analog objects, and (iii) number of people. The analog objects used were circular region models, rectangular region models and length models (cf., Streefland, 1991). The number of analog objects ranged from one to six and the number of people ranged from two to six. The circular region models for the study included: pizza, pancake, pikelet and apple pie. The rectangular region models included: cake and icecream barcake. The length model was represented by the licorice strap (cf., Streefland, 1991).

PROCEDURE

Each interview began with problems using the "pizza" circular region model (cf. Streefland, 1991), then proceeded onto problems using the rectangular region model and the length model. As the literature noted that children have sound informal knowledge of half (Behr et al., 1992; Ball, 1993) and powerful strategies for halving (Pothier & Sawada, 1983), the interview began with problems which produced shares of half and quarter. The sequence of partitioning problems administered to a child after (s)he had attempted these initial problems which generated shares of half or a quarter was not selected a priori. Instead, the interviewer used the child's responses to each problem to inform the selection of problems administered later on in the interview. Each child thus was administered a unique sequence of partitioning problems.

During the course of the interviews, icecream bar and licorice strap problems were presented to all of the children in order to investigate Ball's (1993) contention that partitioning of length models and narrow rectangular region models is easier than the partitioning of circular region models. It was expected that children who had difficulty partitioning one circular region model among three people may successfully partition one length model or one narrow rectangular region model among three people. Problems were included to explore children's partitioning strategies for four, five and six people. Particular attention also was taken of children's partitioning strategies for the wide rectangular region model as Streefland (1991) noted that it supported both one-directional (i.e., vertical or horizontal) and two-directional (i.e., vertical and horizontal) partitioning.

RESULTS AND DISCUSSION

During the course of this study, twelve partitioning strategies emerged. As is indicated in Table 1 below, six of the partitioning strategies used by the children in this study had hitherto been reported in Streefland (1991), English and Halford (1995), Pothier and Sawada (1983), and Lamon (1996). However, six of the strategies used by the children in this study were hitherto unreported in the research literature. Full descriptions of these twelve strategies are provided in Charles (1998) and Charles, Nason and Cooper (submitted).

Table 1
Partitioning Strategies

Hitherto Reported Strategies	Hitherto Unreported Strategies
Partitive quotient foundational strategy *Streefland (1991)* *English & Halford (1995)*	Partitive quotient part-whole strategy
Proceduralised partitive quotient strategy *Streefland (1991)*	Partitive quotient renaming strategy
Horizontal partitioning strategy *Streefland (1991)* *Pothier & Sawada (1983)*	People by objects strategy
Halving the object then halving again and *Streefland (1991)* *Pothier & Sawada (1983)*	Half the objects between half the people again strategy strategy
Half to each person then quarter to each person strategy *Streefland (1991)*	Repeated sizing strategy
Whole to each person then half remaining objects between half the people strategy *Lamon (1996)*	Repeated halving/repeated sizing strategy

We noticed that as each of the children progressed through his or her sequence of partitioning problems, they all sought to find ways of speeding up the process of partitioning and of reducing the load on working memory. As this occurred, we noted the emergence of two very interesting classes of partitioning strategies which we labelled *composite strategies* and *truncated strategies*.

Composite Strategies

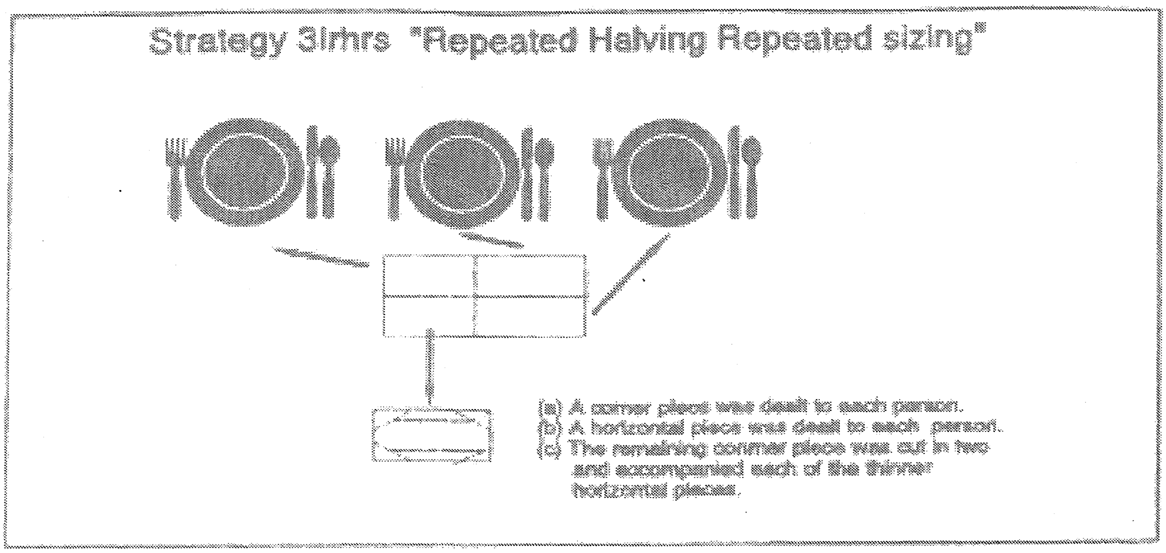
A composite strategy is formed when two or more strategies are combined into a single strategy. In this study, we identified three such strategies: the "whole to each person then half remaining objects between half the people strategy", the "repeated halving/repeated sizing strategy", and the "half to each person then quarter to each person strategy".

The "whole to each person then half remaining objects between half the people strategy" is formed by the combination of the children's strategies for sharing wholes and the "half the objects between half the people strategy" for sharing fractional pieces. Emma employed this strategy to share six pizzas between four people. She shared one whole to each person.

Then she tiered the two "left-over" pizzas, cut them into four halves and shared 1/2 to each person. She quantified each person's share as $1\,{}^{1}/_{2}$.

The "repeated halving/repeated sizing strategy" is a combination of the "halving the object again and again strategy" and the "repeated sizing" strategy. Joshua used this strategy to share a rectangular cake between three people. He cut the cake in half horizontally. Then he cut each half in half and shared 1/4 to each person (i.e., he applied the "halving the object again and again strategy"). He resized the final 1/4 as demonstrated in Figure 2 and shared the pieces as evenly as possible by the attribute of area (i.e., he applied the repeated sizing strategy").

Figure 2
Joshua's Application of "repeated halving/repeated sizing strategy"



The "half to each person then quarter to each person strategy" is a combination of the "half the objects between half the people strategy" and the "partitive quotient foundational strategy". Two of the children in this study, Joshua and Sally, used this strategy when asked to share three pizzas between four people. Joshua partitioned the first two pizzas into halves and shared them fairly between the four people (i.e., he applied the "half the objects between half the people strategy"). Then he partitioned the third and remaining pizza into quarters and shared these quarters between the four people (i.e., he applied the "partitive quotient foundation strategy"). Finally, he quantified the total amount each person received by adding the 1/2 (generated by the application of the "half the objects between half the people strategy") to the 1/4 (generated by the application of the "partitive quotient foundational strategy") to get 3/4. When attempting the same problem, Sally cut the first and second pizza in halves and the third pizza in quarters. She shared 1/2 to each person and then 1/4 to each person. She quantified each share in a similar manner to that used by Joshua. Each of these three composite strategies shared a number of characteristics. First, they were very efficient time-wise at generating equal shares. Second, each composite strategy could only be applied to a small number of partitioning problems. We also noted that the composite strategies only emerged after the child using them had successfully utilised the component strategies on a number of previous partitioning problems. The emergence of the composite strategies thus was highly dependent on the sequence of problems presented to a child during an interview session. If a problem currently being solved could be broken into a sequence of subproblems which were almost identical to problems attempted recently, then the probability of a composite strategy emerging greatly increased.

Truncated Strategies

A truncated strategy is formed by the shortening and/or simplification of an earlier emerging partitioning strategy. We found that the children only truncated strategies after they had successfully applied them a few times. Two types of truncations were identified during the course of this study: *simplification of the physical process of partitioning*, and *cognitive proceduralisation* (cf., Anderson, 1983).

Simplifications of the Physical Process of Partitioning

The "half the objects between half people" strategy is an example of a truncated strategy which greatly simplified the process of partitioning the objects. This strategy involved the following six steps:

Step 1. recognition of the number of people *(n)*

Step 2. recognition of the number of objects *(x)*

Step 3. realisation that halving the objects will generate enough pieces to share between all the people

Step 4. partition the objects into halves

Step 5. sharing the halves between all the people

Step 6. quantification of each share

Joshua, Claudia, Sophie, Thomas, Emma, Sally, Da and Belinda utilised this strategy to share two objects between four people. Thus, instead of partitioning each of the objects into quarters as would have happened if they had applied the "partitive quotient foundational strategy", the children instead partitioned each object into two halves and then distributed the four halves generated between the four people.

Although truncations of this type simplified the physical process of partitioning, they often tended to lead to two types of problems. First, the strategies created by this type of truncation only could be successfully applied to a limited number of partitive quotient fraction problems. Second, the strategies created by truncations of this type, whilst simplifying the physical process of partitioning, more often than not complicated the cognitive processes of accurately quantifying each share and making a conceptual mapping between the concrete activity and the partitive quotient fraction construct. For example, after Claudia had applied the "half the objects between half people strategy" and partitioned the two wholes into halves and distributed the four half objects between four people, she incorrectly assumed that each person had received one part out of four equal parts. It appeared that she had converted the problem from one being represented by region analog models to one being represented by a set analog model consisting of four separate elements. Therefore, she incorrectly quantified each person's share as 1/4 (i.e., one part out of four equal parts) instead of either 1/2 or 2/4 of a pizza.

Cognitive Proceduralisations

The "proceduralised partitive quotient strategy" is an example of a strategy produced by the process of cognitive proceduralisation. This strategy is a truncated, proceduralized version of the "partitive quotient foundational strategy". The "partitive quotient foundational strategy" consists of the following six steps:

Step 1. recognition of number of people *(n)*,

Step 2. generation of fraction name from number of people *(nths)*, Step 3. recognition of relationship between fraction name and number of equal pieces in each whole object *(n equal pieces)*,

Step 4. partitioning of each whole object into equal pieces *(n equal pieces)*,

Step 5. sharing the pieces *(1/n of each object to each person)*, and

Step 6. quantifying each share *(addition of 1/n pieces)*

In the "proceduralised partitive quotient strategy", the foundational strategy is truncated down to the following three steps:

Step 1. recognition of number of people *(n)*

Step 2. recognition of number of analog objects *(x)*

Step 3. quantification of each person's share as x/n

This proceduralisation reduces the load on working memory involved in the solution of a partitive quotient fraction problem. This proceduralisation is also indicative of progress towards the automatisation of the process of abstractly generating a numerical solution to partitive quotient fraction problems. For example, when Caitlin and Sally used this strategy, they no longer felt the need to actually physically partition and share the objects. When Mark and Joshua executed this proceduralised strategy, they also indicated that they were well on the way to automatising the process of abstractly generating a numerical solution to partitioning problems. They executed all three steps of the strategy and then utilised the concrete analogs to confirm the correctness of their solutions. For example, when sharing three pizzas between four people, Joshua and Mark did not cut each pizza into quarters but instead they cut 1/4 from each of three pizzas leaving 3/4 of each of the three pizzas intact. They then placed the three 1/4 pieces together to form the fourth share.

Cognitive proceduralisation truncations such as the "proceduralised partitive quotient strategy" did not cause the two types of problems endemic with the first type of truncated strategies. Instead, the cognitive proceduralisation truncations resulted in the development of more efficient, automatized strategies which could be successfully applied to wide varieties of partitive quotient fraction problems.

CONCLUSION

In this paper, we reported on two subsets of partitioning strategies utilised by a sample of twelve year three students when they were asked to solve sequences of "realistic" partitive quotient fraction problems: *composite strategies* and *truncated strategies*. These two types of partitioning strategies emerged as each of the children progressed through his or her sequence of partitioning problems and began to seek ways of making the process of partitioning more efficient in terms of generating equal shares, reducing the number of steps and reducing the load on working memory.

The composite strategies were very efficient in terms of generating equal shares, particularly in the context of specific partitioning problems. However, they could not be applied generally to all of the partitioning problems. Furthermore, their efficacy in terms of facilitating the abstraction of the partitive quotient fraction construct from the physical activity of partitioning objects was greatly influenced by the abstraction-ability of the component strategies. If the component strategies facilitated the generation of fair shares, the accurate quantification of shares and a conceptual mapping between the physical activity of partitioning and the partitive quotient fraction construct, then the composite strategy was efficacious in terms of facilitating the abstraction of the partitive quotient fraction construct.

Two types of truncated strategies were identified: *simplification of the physical process of partitioning*, and *cognitive proceduralisation*. The former type of truncated strategy, whilst simplifying the physical process of partitioning, more often than not complicated the cognitive processes of accurately quantifying each share and making a conceptual mapping between the concrete activity and the partitive quotient fraction construct. In contrast to this, the latter type of truncated strategies not only generated fair shares, reduced the number of steps, but also facilitated the process of abstracting the partitive quotient fraction construct. Furthermore, the cognitive proceduralisation strategies greatly reduced the load on working memory.

A number of implications for instruction can be drawn from these findings. First, is that the emergence of composite and truncated strategies indicate that children are actively constructing knowledge and are not merely going through the process of completing the partitioning problems. Second, is that children can follow a number of developmental paths as they seek ways of making the process of partitioning more efficient in terms of generating equal shares, reducing the number of steps and reducing the load on working memory. They can construct composite strategies, truncated strategies, and/or combinations of these types of strategies.

However, some of these types of strategies, even if they lead to the successful completion of sharing tasks, do not assist children to abstract the partitive quotient fraction construct from the physical activity of partitioning. Therefore, teachers need to be aware of the strengths and limitations of the composite and simplification of physical process truncated strategies and if possible, actively facilitate the construction of cognitive proceduralised truncated strategies. This can be done by: (i) the careful sequencing of partitive quotient fraction problems in the context of circular region models, rectangular region models, and length models to reinforce the conceptual mapping of the partitive quotient fraction construct from the concrete activities and (ii) the presentation of more complex (interrelated by number of people and number of objects) tasks to generalise the partitive quotient fraction construct from the concrete activities such that x objects shared among n people results in a share of $^x/_n$.

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